Vertical Trajectory Calculation

For a sphere of radius
r = 50 cm and density p = 860 kg/m^3 = 0.86 gm/cm^3

the mass is m = 450294.9 gm.

Assume the object has a drag coefficient C = 0.5 (the default value is C = 0.5 for a sphere) and is falling through air with density p* = 1.29 kg/m^3 (the default value is 1.29 kg/m^3).

Under these conditions, it's terminal velocity will be
v_t = 131.99326 m/s
v_t = 475.17575 km/hr
v_t = 295.25573 mi/hr

and its characteristic time is
t = 13.468700 s.

If this sphere is launched vertically with a velocity of
v_0 = 80 m/s = 262.46719 ft/s

then on the way up at height y_1 = 100 m = 328.08398 ft
it will have velocity v_1 = 62.171153 m/s = 203.97360 ft/s

It will reach a peak height y_{peak} = 278.10873 m = 912.43027 ft
at time t_{peak} = 7.3389135 s

On the way down at height y_2 = 100 m = 328.08398 ft
it will have velocity v_2 = 56.244314 m/s = 184.52859 ft/s

It will reach the ground at velocity
v_{impact} = 68.414888 m/s = 224.45829 ft/s = 246.29359 km/hr = 153.03726 mi/hr
at time t_{impact} = 7.7315906 s.

For comparison, if there were no air friction the projectile would have reached height
h = 326.53061 m = 1071.2946 ft
and would impact with the original launch velocity at time t = 16.326530 s.

Air drag can be expressed in the form

$$f_{drag} = -\frac{1}{2} C\rho^* A v^2$$

and it is equal to the weight of the object at the terminal velocity

$$v_t = \sqrt{\frac{2mg}{C\rho^* A}}$$

The characteristic time is

$$\tau = \frac{v_t}{g}$$

ρ^* = air density = 1.29 kg/m^3 nominally.

C = 0.5 for a sphere