Vertical Trajectory Calculation

For a sphere of radius
r = 100 cm and density p = 107.3999 kg/m^3 = 0.1074 gm/cm^3

the mass is m = 449876.0 gm.

Assume the object has a drag coefficient C = 0.5 (the default value is C = 0.5 for a sphere) and is falling through air with density p* = 1.29 kg/m^3 (the default value is 1.29 kg/m^3).

Under these conditions, it's terminal velocity will be
v_t = 65.96592 m/s
v_t = 237.4773 km/hr
v_t = 147.5591 mi/hr

and its characteristic time is
t = 6.731217 s.

If this sphere is launched vertically with a velocity of
v_0 = 80 m/s = 262.4671 ft/s

then on the way up at height y_1 = 100 m = 328.0839 ft
it will have velocity v_1 = 50.01085 m/s = 164.0776 ft/s

It will reach a peak height y_{peak} = 200.8183 m = 658.8529 ft
at time t_{peak} = 5.931883 s

On the way down at height y_2 = 100 m = 328.0839 ft
it will have velocity v_2 = 39.85259 m/s = 130.7499 ft/s

It will reach the ground at velocity
v_{impact} = 50.89499 m/s = 166.9783 ft/s = 183.2219 km/hr = 113.8470 mi/hr
at time t_{impact} = 6.893494 s.

For comparison, if there were no air friction the projectile would have reached height
h = 326.5306 m = 1071.294 ft
and would impact with the original launch velocity at time t = 16.32653 s.

Air drag can be expressed in the form

$$f_{drag} = -\frac{1}{2} C\rho^* A v^2$$

and it is equal to the weight of the object at the terminal velocity

$$v_t = \sqrt{\frac{2mg}{C\rho^* A}}$$

The characteristic time is

$$\tau = \frac{v_t}{g}$$

ρ^* = air density = 1.29 kg/m^3 nominally.

C = 0.5 for a sphere